A traffic model for pedestrian and its comparison with experimental data

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Workshop on Pedestrian Traffic Flows
**Objective:** Modeling pedestrian motion in a corridor.
Different types of models:

- Cellular automaton
- Differential equations:
  \[
  \frac{dx_i}{dt} = v_i, \quad \frac{dv_i}{dt} = F_i
  \]
- Macroscopic model:
  \[
  \partial_t \rho + \partial_x f(\rho) = 0
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Introduction

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Outline

1. A model for pedestrian traffic flows
2. Real experiments
3. Experiments Vs Model
The model

We propose the following model:

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\begin{align*}
\partial_t \rho_B + \partial_x f(\rho_B, \rho_R) &= 0 \delta \partial^2_x \rho_B \\
\partial_t \rho_R - \partial_x f(\rho_R, \rho_B) &= 0 \delta \partial^2_x \rho_R
\end{align*}
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The flux function $f$ has to satisfy:

- $f(x, y)$ is decreasing in $y$.
- $f(x, y)$ has a "bell-shape" in $x$.

Ex. $f(x, y) = \begin{cases} 
  x(1 - x - y) & \text{if } 0 \leq x + y \leq 1 \\
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Properties of the model

**Thm.** The model (1) is *hyperbolic* if and only if:

\[
\Delta = (\partial_x f + \partial_x \tilde{f})^2 - 4\partial_y f \partial_y \tilde{f} \geq 0,
\]

where \( f = f(\rho_B, \rho_R) \) and \( \tilde{f} = f(\rho_R, \rho_B) \).

**Ex.** For \( f(x, y) = x \frac{g(x+y)}{x+y} \) with \( g \) a “bell-function”, we have:
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Numerical schemes

We use a *central-scheme* method to solve the system (1):

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\frac{U_{i}^{n+1} - U_{i}^{n}}{\Delta t} + \frac{1}{\Delta x} (F_{i+1/2} - F_{i-1/2}) = \delta \frac{U_{i-1}^{n} - 2U_{i}^{n} + U_{i+1}^{n}}{\Delta x^2}.
\]

Here, \( U_{i}^{n} = (\rho_B, \rho_R)^T \) and \( F(\rho_B, \rho_R) = (f(\rho_B, \rho_R), -f(\rho_R, \rho_B))^T \).

\( F_{i+1/2} \) denotes the numerical flux at \( x_{i+1/2} \) defined as:

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F_{i+1/2} = \frac{F(U^L_{i+1/2}) + F(U^R_{i+1/2})}{2} - a_{i+1/2} \frac{U^R_{i+1/2} - U^L_{i+1/2}}{2},
\]

with \( a_{i+1/2} \) the maximum eigenvalues at \( x_i \) and \( x_{i+1} \), \( U^L_{i+1/2} \) and \( U^R_{i+1/2} \) are (resp.) the left and right value of \( U \) at \( x_{i+1/2} \) (MUSCL scheme).
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Numerical simulations

For the initial condition, we choose a stationary state (e.g. \( \rho_R \) and \( \rho_B \) constant) perturbed by some noise. We use periodic boundary conditions.

First, we choose \((\rho_B, \rho_R)\) in the hyperbolic region:
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![Graph showing the initial condition propagating and diffusing](image)

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Then, we initiate with a stationary state in the hyperbolic region:

We observe the apparition of clusters. Inside the cluster, the total density is around the maximum 1, thus the flux is zero and the clusters are immobile.
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![Graph showing (rhoB, rhoR) at time t = 100.00](image)

We observe the apparition of *clusters*. Inside the cluster, the total density is around the maximum 1, thus the flux is zero and the clusters are immobile.
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Real experiments

**Objective:** We would like to compare the model with real experimental data.

Set-up for the experiments

Collected data
Estimation of the densities

We only consider the angle position $\theta_i$ of the pedestrian:
Results

We plot the densities \((\rho_B, \rho_R)\) in the experiments over time:

- Formation of bands.
- The speed of the bands is lower when the density is higher.
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Estimation of the flux

To compare the experiences with the model, we need first to estimate the flux function $f(\rho_B, \rho_R)$.

- $f$ is decreasing with $\rho_R$.
- $f$ seems to saturate when $\rho = \rho_B + \rho_B \approx 2.5$.

Pbm: No data for $\rho \geq 2.5$. $\Rightarrow$ We extend by a “bell-shape” function:

$$f(\rho_B, \rho_R) = \rho_B (1.29 - .26 \rho_B - .24 \rho_R) \approx \frac{1}{4} \rho_B \cdot (5 - \rho).$$

Remark. 5 ped/m² is usually considered as the maximum density.
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We run the model with the same initial condition as in the experiments:

- The model captures well the formation of traveling bands. These bands are also slower when the density is higher.
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- However, the model seems to diffuse more.
For one experiment, the model hits the zone of non-hyperbolicity:
Adding a small amount of diffusion ($\delta = .05$) prevents the formation of clusters:
Conclusion

- We have proposed a simple model to describe the motion of pedestrian in a corridor.
- This model leads to the formation of clusters when the total density is high.
- It also captures some features of the experimental data (traveling bands).

In the future, we would like to:
- find an appropriate quantifier to measure the agreement between the model and the experiments.
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