A traffic model for pedestrian and its comparison with experimental data

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Workshop on Pedestrian Traffic Flows

Introduction

Objective: Modeling pedestrian motion in a corridor.

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Different types of models:

- Cellular automaton
- Differential equations:

$$\begin{aligned} \frac{dx_i}{dt} &= v_i \\ \frac{dv_i}{dt} &= F_i \end{aligned}$$

- Macroscopic model:

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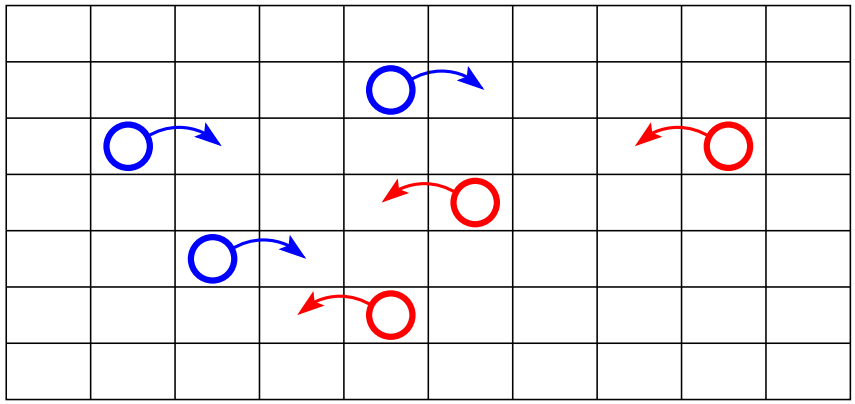

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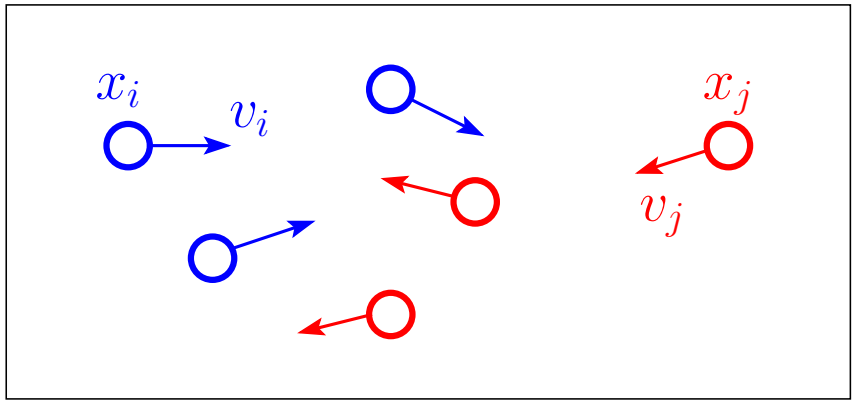

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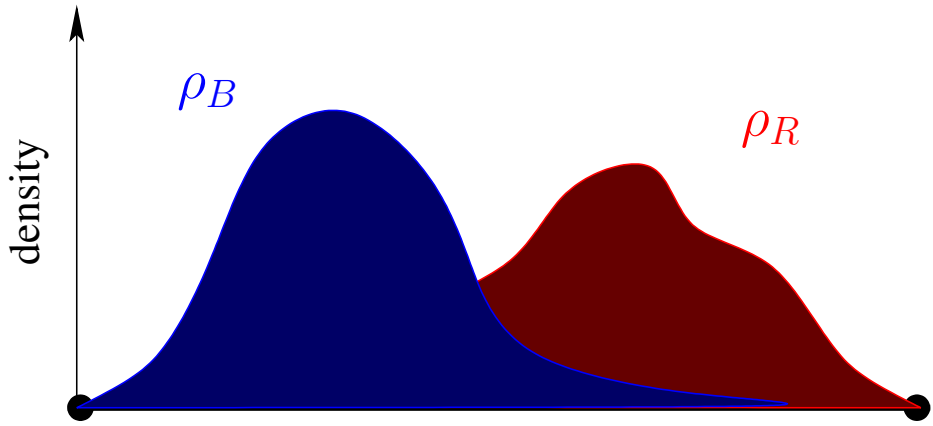

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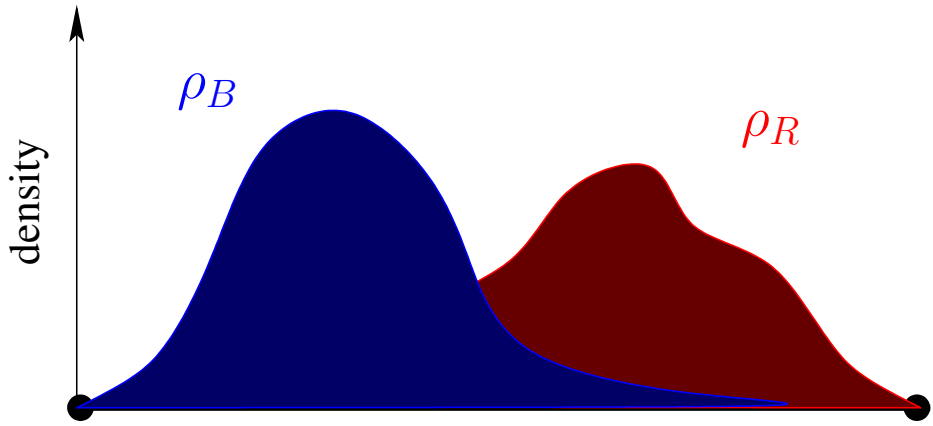

Outline

The model

We propose the following model:

$$\begin{aligned} \partial_t \rho_B + \partial_x f(\rho_B, \rho_R) &= 0 \delta \, \partial_x^2 \rho_B \\ \partial_t \rho_R - \partial_x f(\rho_R, \rho_B) &= 0 \delta \, \partial_x^2 \rho_R \end{aligned} \tag{1}$$

The flux function f has to satisfy:

- $f(x, y)$ is decreasing in y.
- $f(x, y)$ has a "bell-shape" in x.

Ex. $f(x, y) = \begin{cases} x(1 - x - y) & \text{if } 0 \leq x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$

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Properties of the model

Thm. The model (1) is *hyperbolic* if and only if:

$$\Delta = (\partial_x f + \partial_x \widetilde{f})^2 - 4\partial_y f \partial_y \widetilde{f} \geq 0,$$

where $f = f(\rho_B, \rho_R)$ and $\widetilde{f} = f(\rho_R, \rho_B)$.

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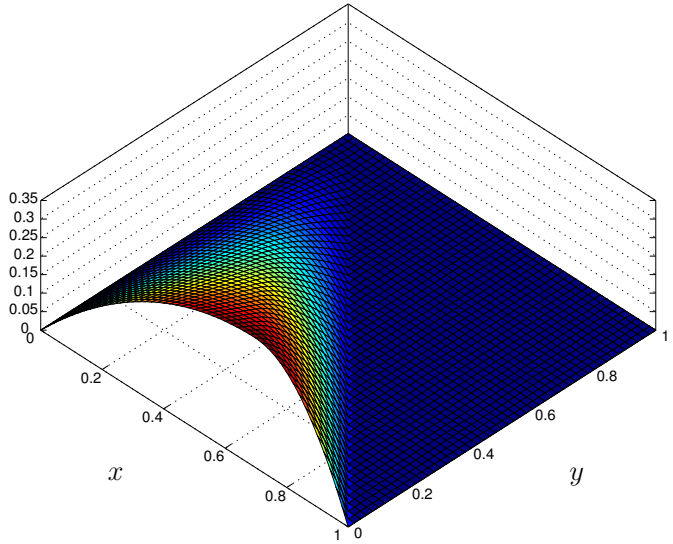

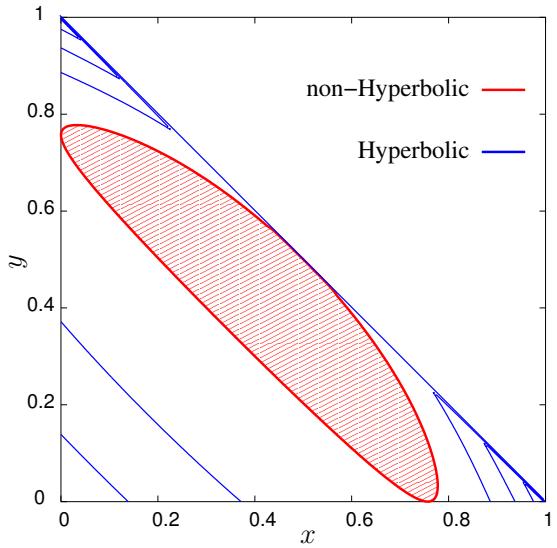

Numerical schemes

We use a *central-scheme* method to solve the system (1):

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + \frac{1}{\Delta x}\left(F_{i+1/2} - F_{i-1/2}\right) = \delta\,\frac{U_{i-1}^n - 2U_i^n + U_{i+1}^n}{\Delta x^2}.$$

Here, $U_i^n = (\rho_B, \rho_R)^T$ and $F(\rho_B, \rho_R) = (f(\rho_B, \rho_R), -f(\rho_R, \rho_B))^T$.

$F_{i+1/2}$ denotes the numerical flux at $x_{i+1/2}$ defined as:

$$F_{i+1/2} = \frac{F(U_{i+1/2}^L) + F(U_{i+1/2}^R)}{2} - a_{i+1/2}\frac{U_{i+1/2}^R - U_{i+1/2}^L}{2},$$

with $a_{i+1/2}$ the maximum eigenvalues at x_i and x_{i+1}, $U_{i+1/2}^L$ and $U_{i+1/2}^R$ are (resp.) the left and right value of U at $x_{i+1/2}$ (MUSCL scheme).

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Numerical simulations

For the initial condition, we choose a stationary state (e.g. ρ_R and ρ_B constant) perturbed by some noise. We use periodic boundary conditions.

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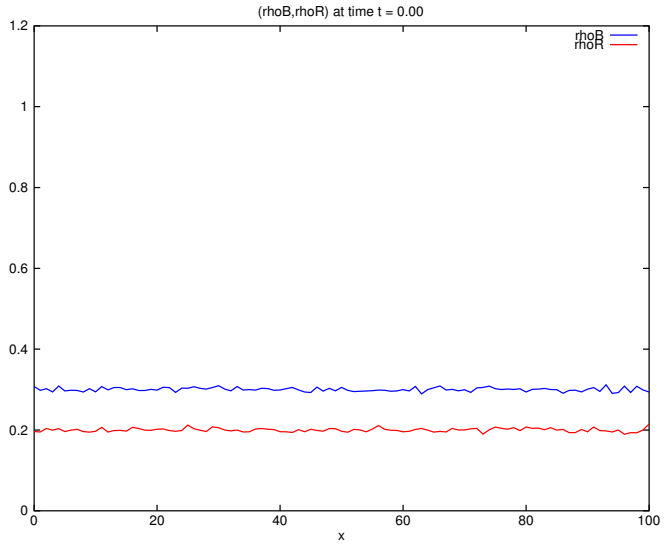

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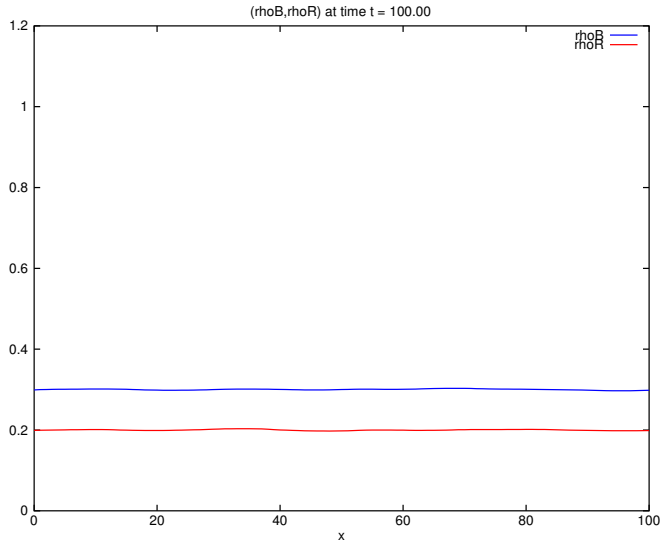

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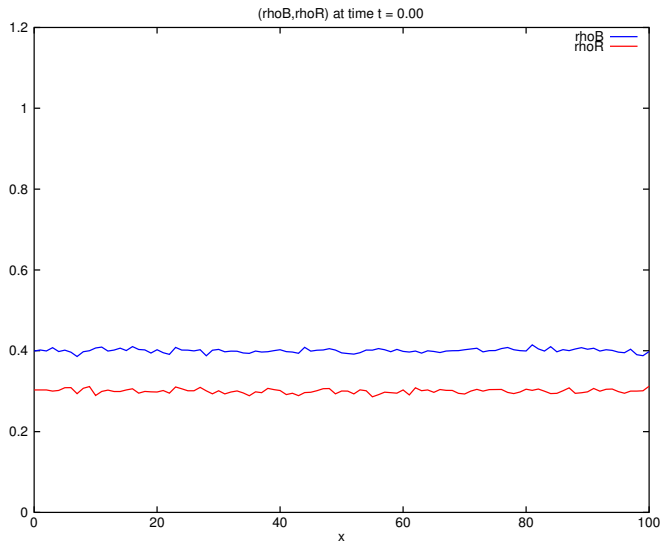

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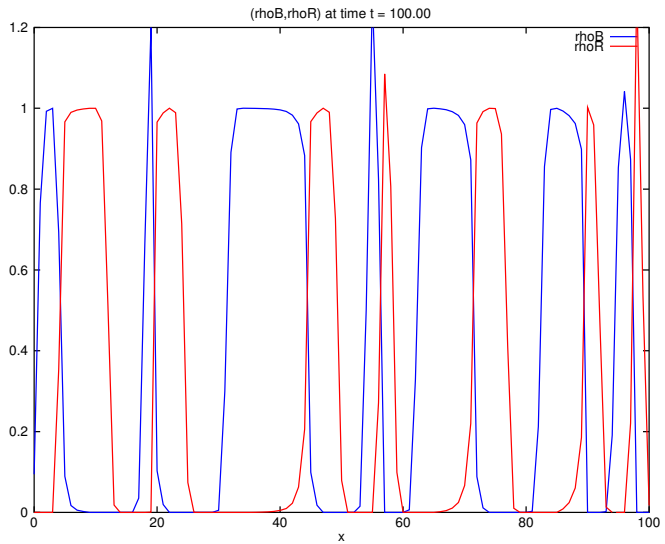

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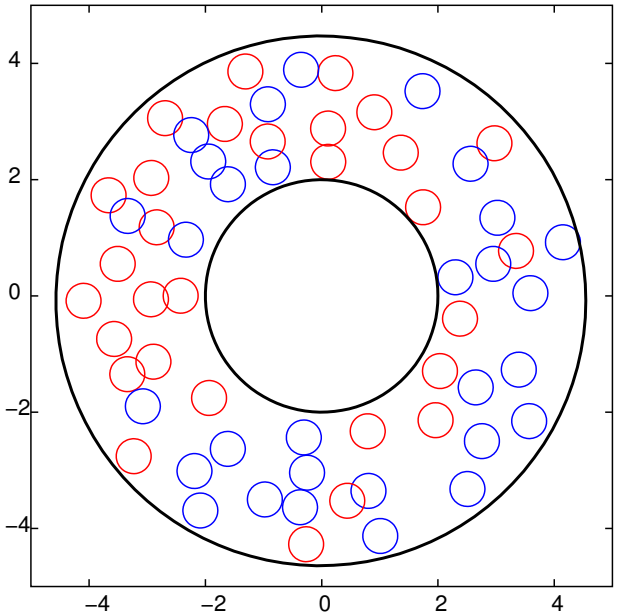
Outline

Real experiments

Objective: We would like to compare the model with real experimental data.

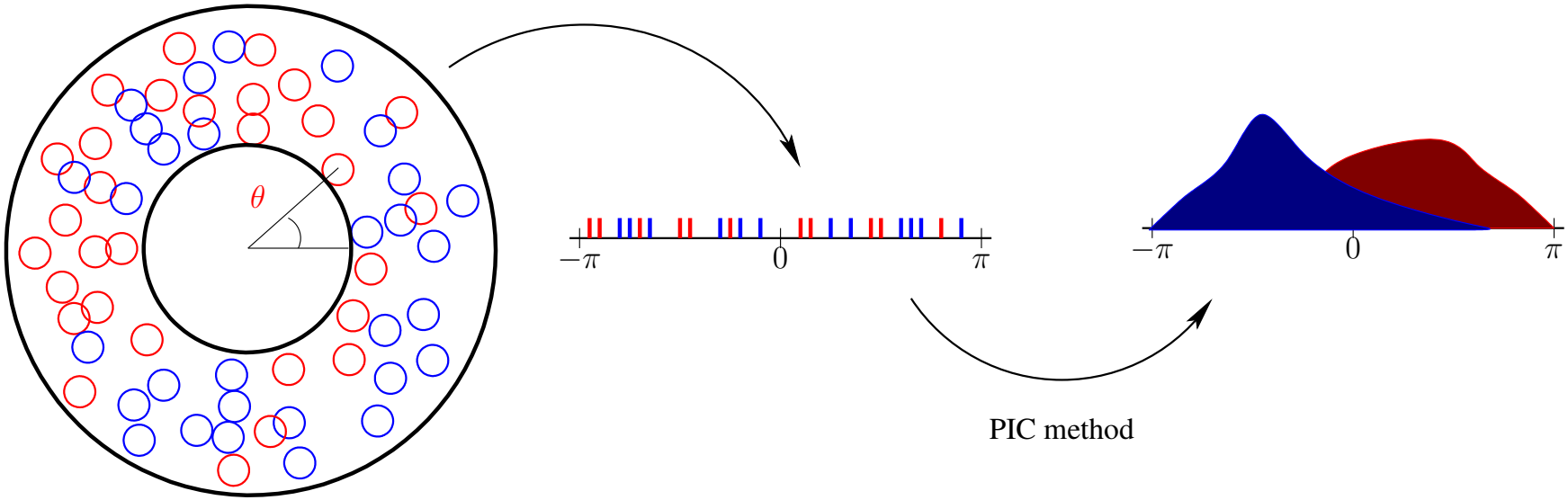
Set-up for the experiments



Collected data

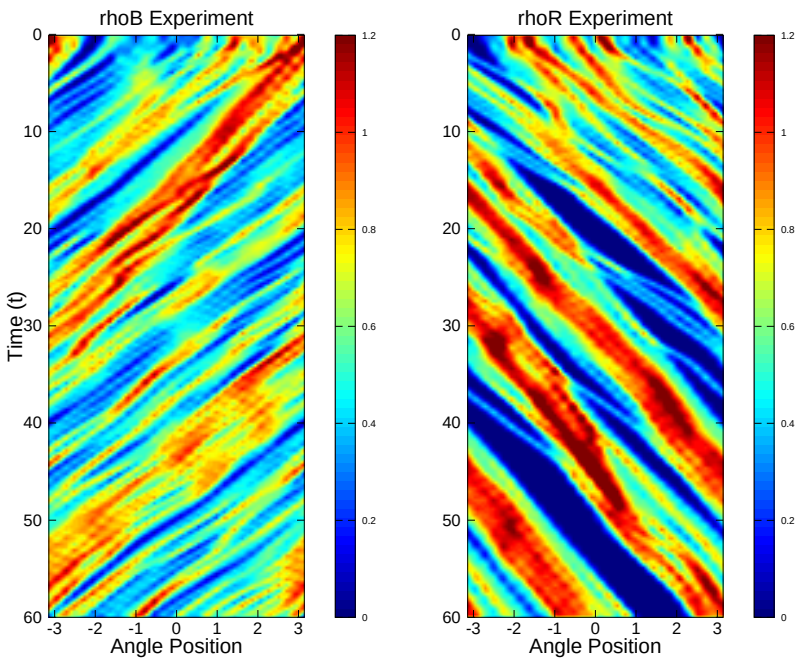
Estimation of the densities

We only consider the angle position θ_i of the pedestrian:



Results

We plot the densities (ρ_B, ρ_R) in the experiments over time:



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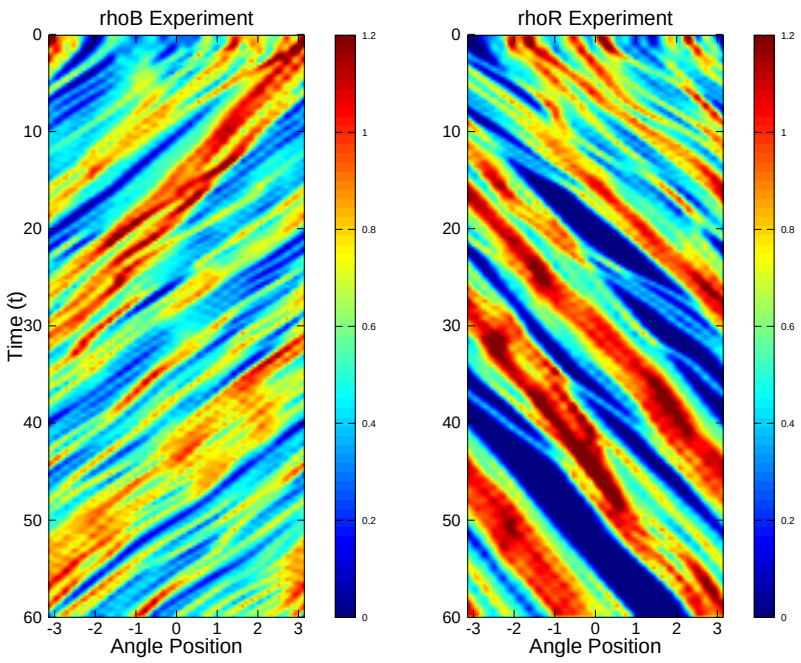

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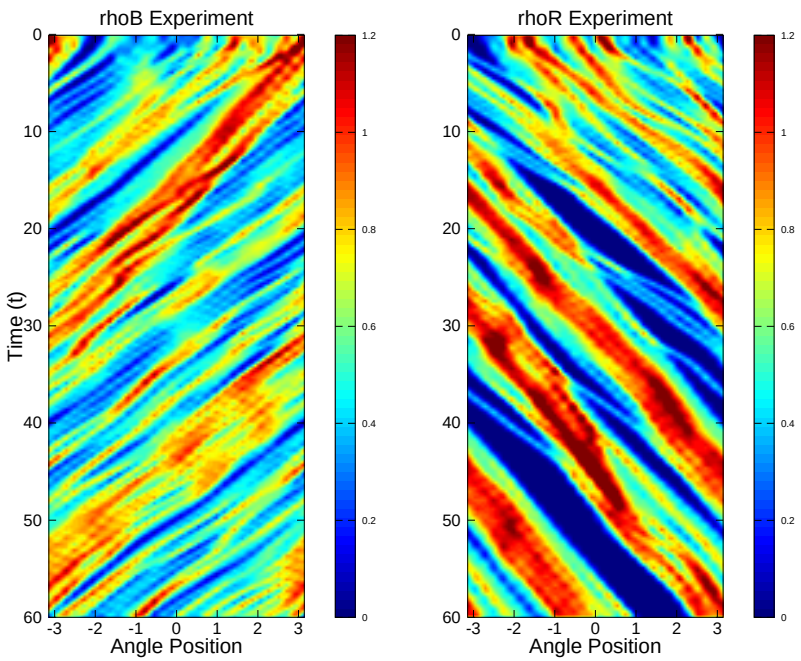

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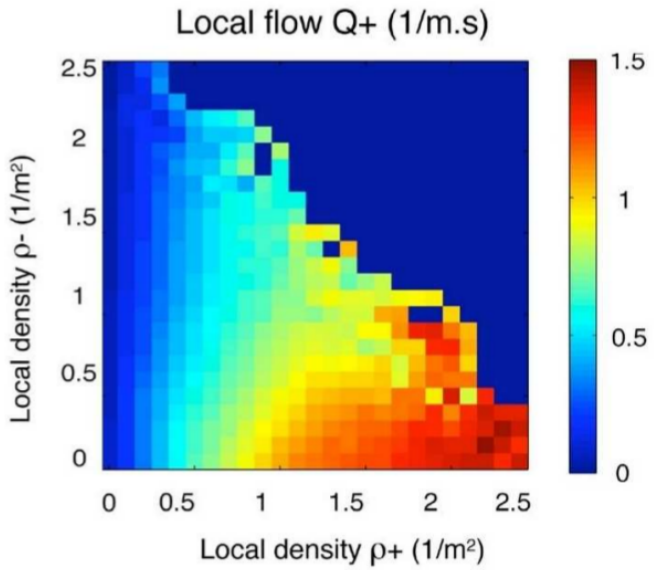

- f is decreasing with ρ_R.
- f seems to *saturate* when $\rho = \rho_B + \rho_B \approx 2.5$.

Pbm: No data for $\rho \geq 2.5$. $\Rightarrow$ We extend by a "bell-shape" function:

$$f(\rho_B, \rho_R) = \rho_B(1.29 - .26\rho_B - .24\rho_R) \approx \frac{1}{4}\rho_B \cdot (5 - \rho).$$

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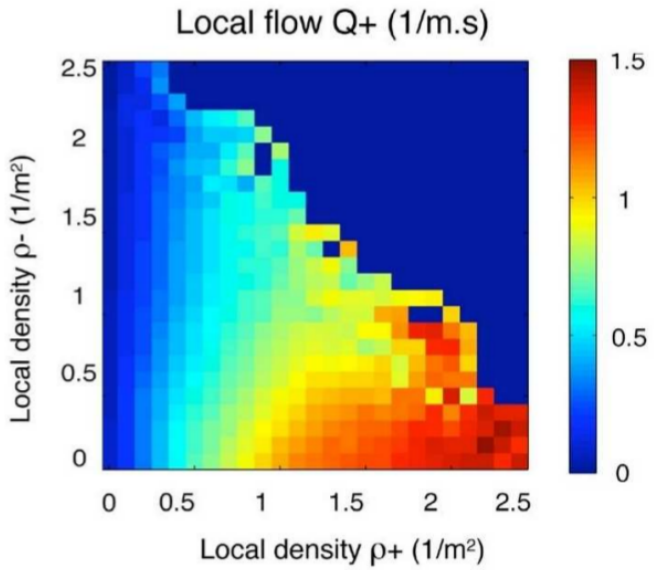

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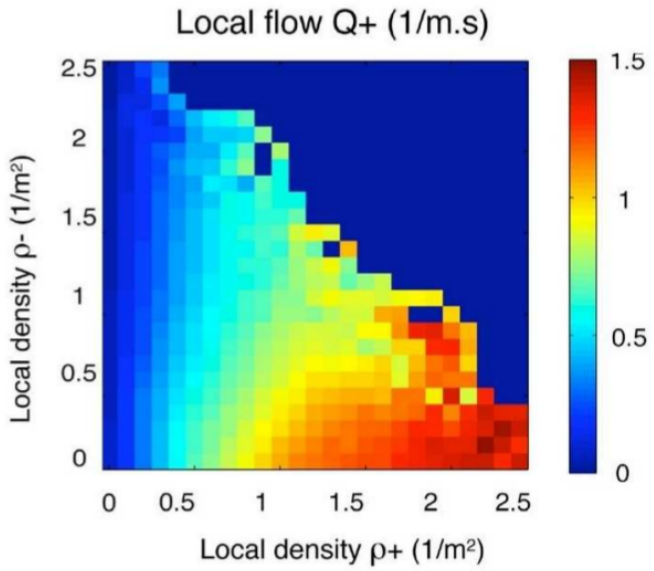

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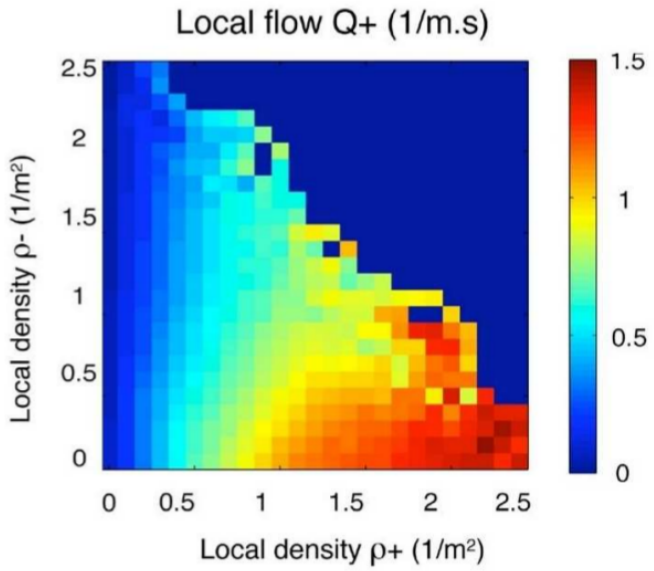

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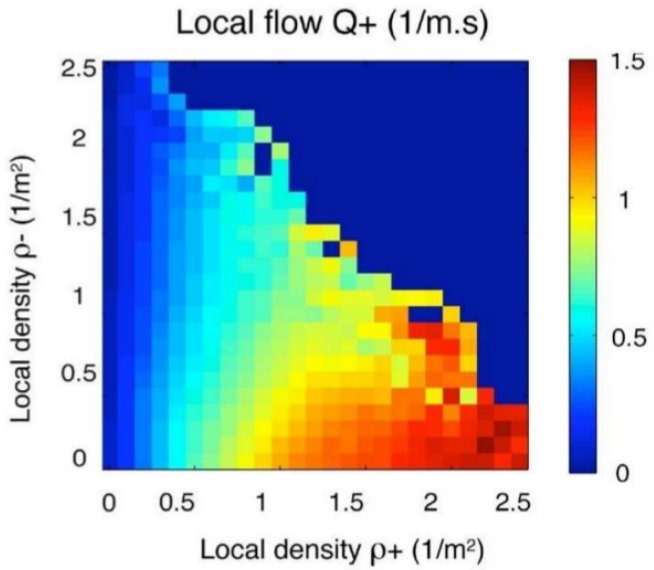

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- However, the model seems to diffuse more.

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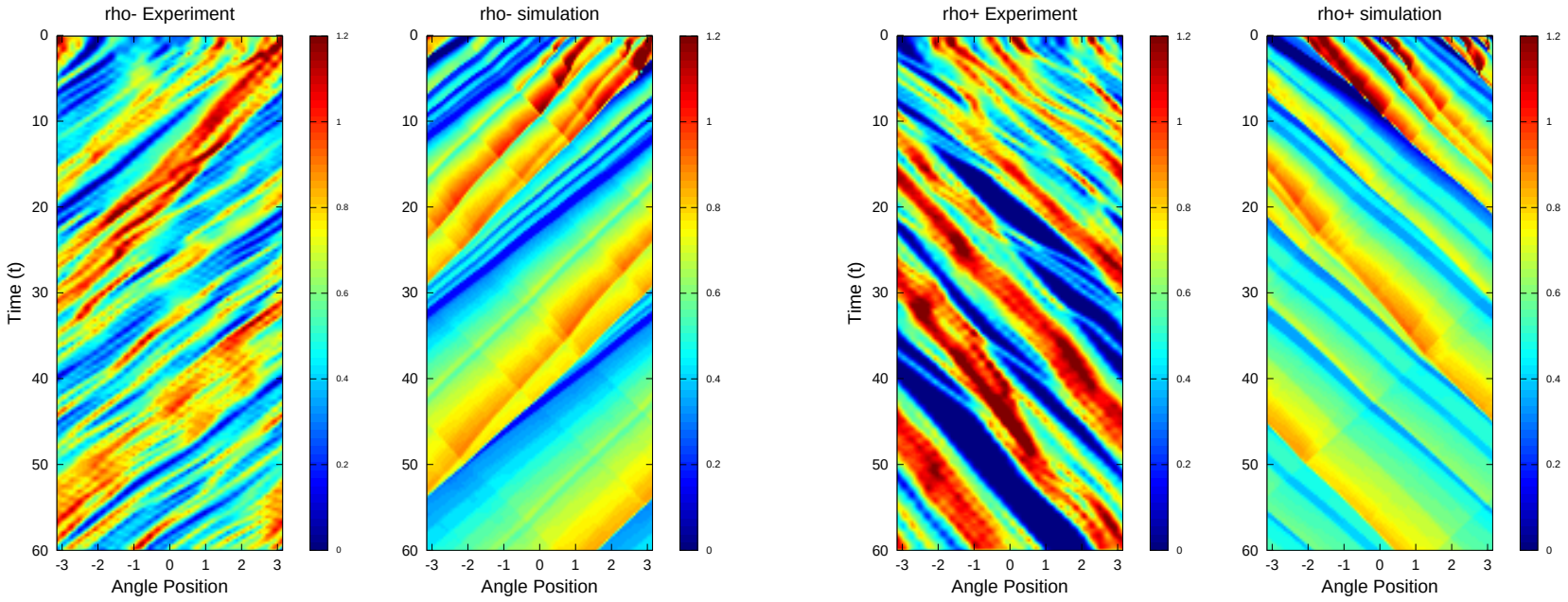

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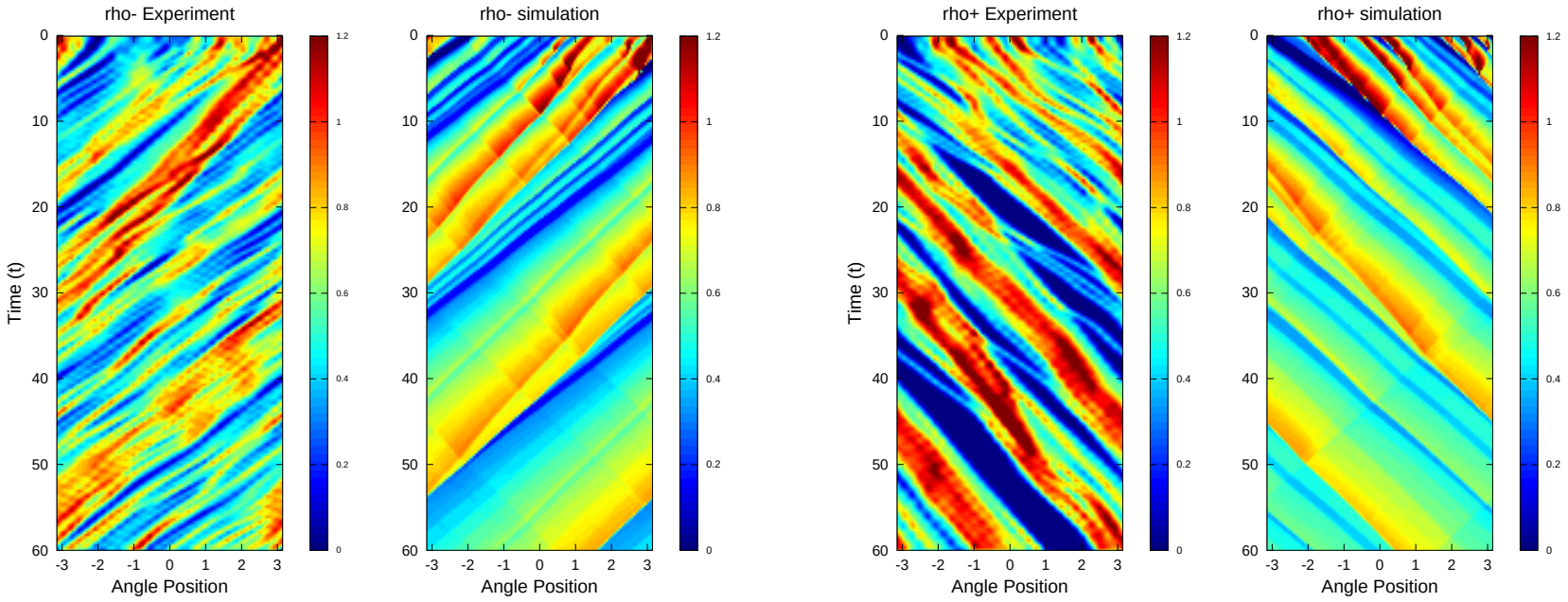

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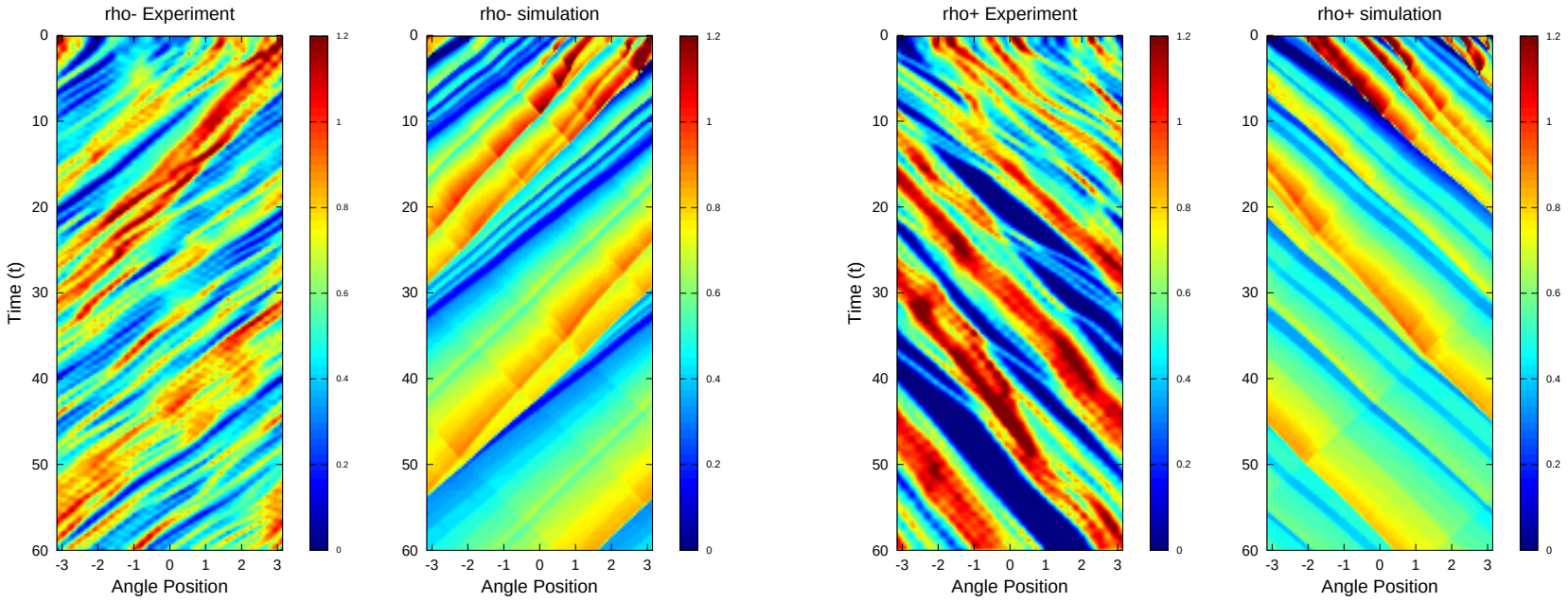

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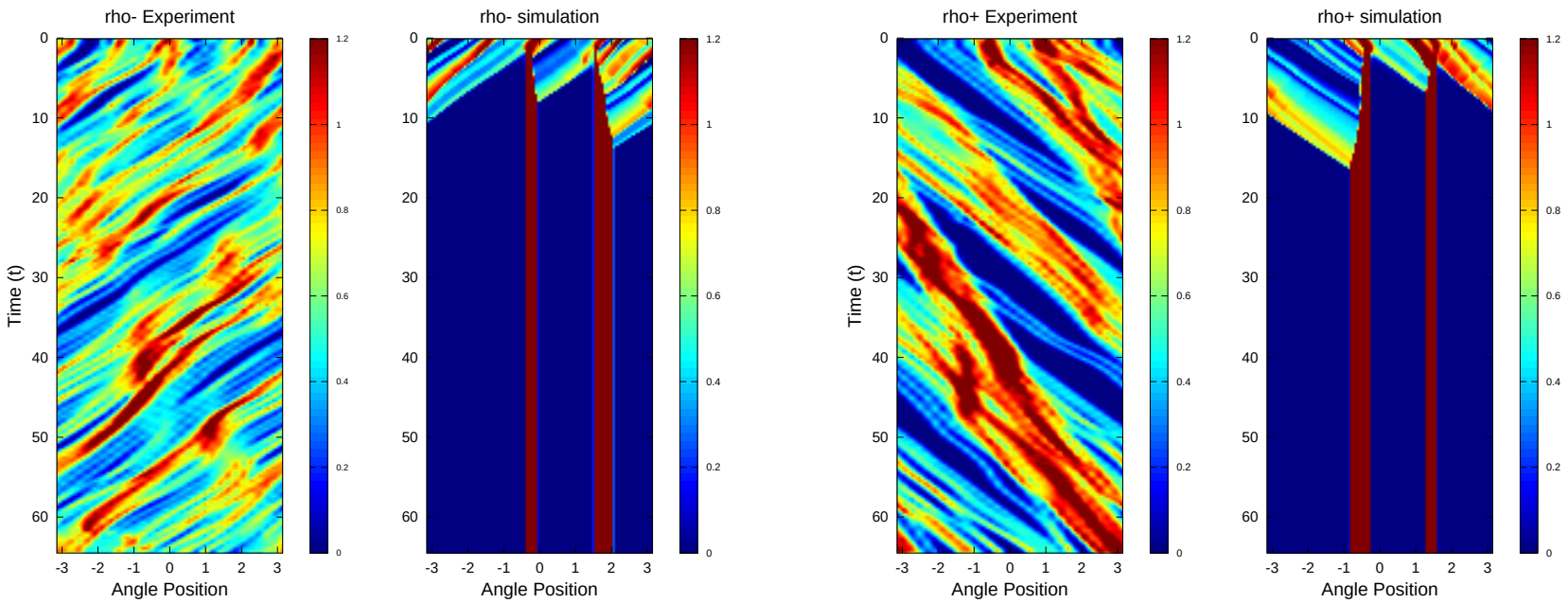
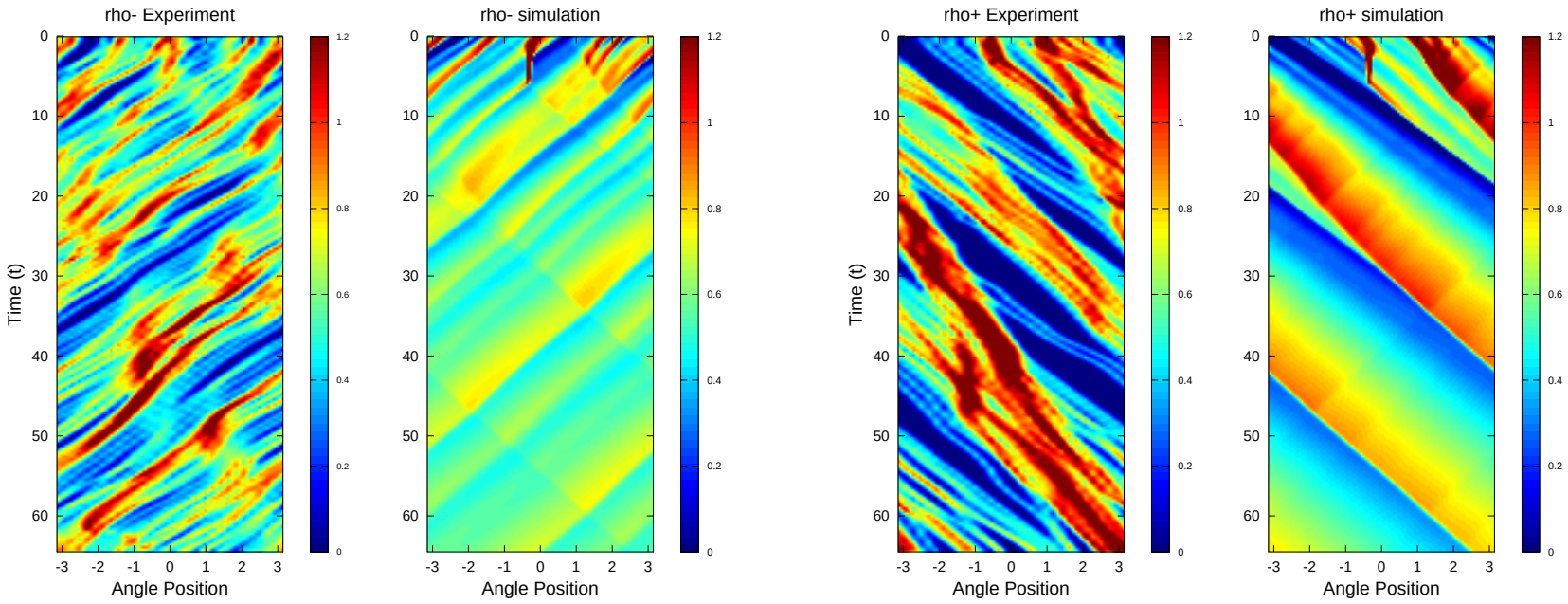
For one experiment, the model hits the zone of non-hyperbolicity:



Adding a small amount of diffusion ($\delta = .05$) prevents the formation of clusters:



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