Ex 1. Use the finite difference method to solve the following (linear) problem:

\[
\begin{cases}
    y'' = 4(y - x) \\
    y(0) = 0, \\ y(1) = 2.
\end{cases}
\]

Find the accuracy of the method.

*Hint: the exact solution is* \( y(x) = \frac{e^2}{e^4-1}(e^{2x} - e^{-2x}) + x \).

Ex 2. Use a finite difference method combined with the Newton method to solve:

\[
\begin{cases}
    y'' = \cos y \\
    y(0) = y(1) = 0.
\end{cases}
\]

[Extra] Find the accuracy of the scheme.

*Hint: find the solutions for different \( \Delta x = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4} \) (5 iterations of the Newton’s method is usually enough). Compare the solutions with the one having the higher accuracy.*

Ex 3. Use the non-linear finite difference method to solve:

\[
\begin{cases}
    y'' = y^3 - yy' \\
    y(1) = \frac{1}{2}, \\ y(2) = \frac{1}{3}.
\end{cases}
\]

Compare the solution with the one given by the shooting method (accuracy? computation time?).