

MAT 425: Homework 8 (03/30)

Ex 1. Use the finite difference method to solve the following (linear) problem:

$$\begin{cases} y'' = 4(y - x) \\ y(0) = 0, y(1) = 2. \end{cases}$$

Find the accuracy of the method.

Hint: the exact solution is $y(x) = \frac{e^2}{e^4-1}(e^{2x} - e^{-2x}) + x$.

Ex 2. Use the finite difference method combined with the Newton method to solve:

$$\begin{cases} y'' = \cos y \\ y(0) = y(1) = 0. \end{cases}$$

[Extra] Find the accuracy of the scheme.

Hint: find the solutions for different $\Delta x = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ (5 iterations of the Newton's method is usually enough). Compare the solutions with the one having the higher accuracy.

Ex 3. Use the non-linear finite difference method to solve:

$$\begin{cases} y'' = y^3 - yy' \\ y(1) = \frac{1}{2}, y(2) = \frac{1}{3}. \end{cases}$$

Compare the solution with the one given by the shooting method (accuracy? computation time?).