MAT 475: Solution homework 8 (11/23)

1 Chapter 9

Ex 2. [3pts]

To draw the phase portrait, we notice that that the homocline $x' = 0$ are given by $x = \pm 1$. Thus, we deduce 4 solutions. In the case $a = 0$, we can go even further (see figure 1). The sign of a dictates the behavior of the solution starting at the origin.

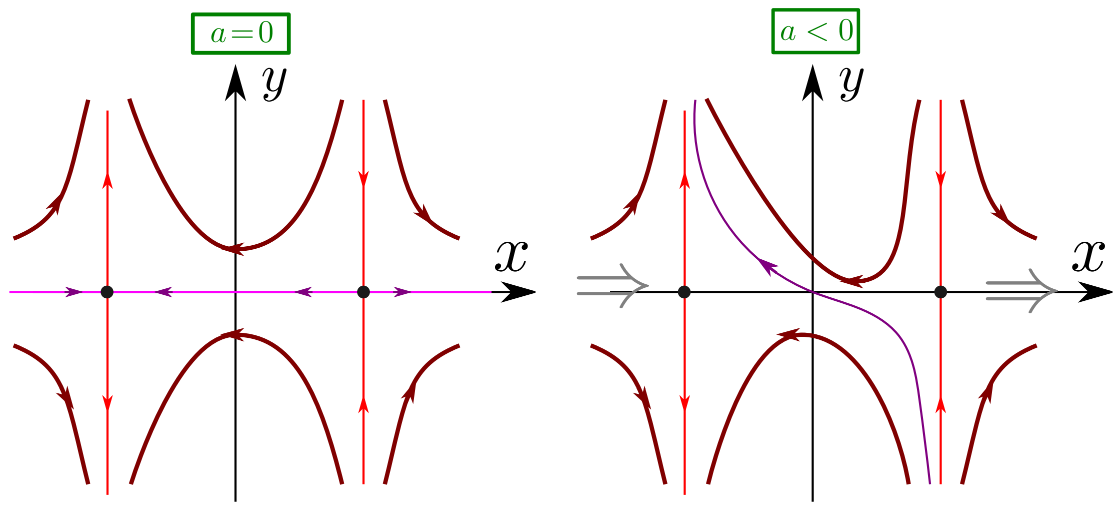

Figure 1: Phase portrait for **Ex.2** with $a = 0$ (left figure) and $a < 0$ (right figure).

Ex 6. [3pts]

Take $V = x^2 + 2y^2$. We have

$$V' = x^2(-4 + 4y) + y^2(-4 + 2x).$$

Thus, for $y < 1$ and $x < 2$, $V' < 0$ outside the origin. In particular, for $\delta = 1$, we have $V' < 0$ for all solution $\mathbf{x}(t)$ inside the unit ball (i.e. $|\mathbf{x}(t)| < 1$). Therefore, the unit ball is contained in the basin of attraction of the origin.

Ex 7. [4pts]

a) Level curves are ellipses: $x^2 + 2y^2 = C$ (see Fig.). Equilibrium $\mathbf{x}_* = (0, 0)$ asymp. stable.

b) Level curves are hyperboles centred at $(1, 2)$: $(x - 1)^2 - (y - 2)^2 = C$. Equilibrium $\mathbf{x}_* = (1, 2)$ saddle.

c) Equilibria are $(n\pi, 0)$ and there are saddle points.

d) Level curves are (rotated) ellipses. Equilibria $(-4/3, -2/3)$ asymp. stable.

e) Level curves are (2D) parabolas. No equilibrium.

f)