MAT 423: Homework 2 (09/05)

Ex 1.

Implement the Newton method and the secant method with the numerical software of your choice.

Ex 2. [Convergence rate for the bisection, Newton and secant methods]

The goal is to estimate how fast each algorithm converges. As an illustration, we try to find a zero x_* of the function:

$$f(x) = x\mathrm{e}^x - 1.$$

a) Estimate the zero x_* with machine precision 10^{-16}. We denote x_{ref} this estimation.

b) Compute the errors $e_n = |x_n - x_{\text{ref}}|$ for the bisection, Newton and secant methods using the initial points of your choice.
Plot the evolution of the error for the three methods, i.e number of iterations on the x-axis and errors e_n on the y-axis (log-scale recommended).

c^*) Estimate the decay rate, i.e. $e_{n+1} \approx C \cdot e_n^\alpha$, for each method.

Hint: use a linear regression to estimate $\ln e_{n+1} \approx c_0 + c_1 \ln e_n$.

Ex 3. [Fixed point theorem]

Consider the function $\varphi(x) = \sqrt{x+1}$.

a) Show that φ has a unique fixed point x_* on $I = [0, \infty)$.

b) Find an approximation of x_* with accuracy 10^{-4}.
Hint: consider $J = [1, 2]$.

Ex 4.

Consider the function $\varphi(x) = \sqrt{x^2+1}$ and the interval $I = [0, \infty)$.

a) Show that for any $x, y \in I$, we have: $|\varphi(x) - \varphi(y)| < |x - y|$.

b) Let $x_0 = 0$ and $x_{n+1} = \varphi(x_n)$. Does the sequence converge?

c) Show that φ does not have a fixed point on I.
Does that contradict the *fixed-point theorem*?