MAT 423: Homework 4 (09/19)

Ex 1.

a) Find the LU decomposition of the matrix:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -4 & 3 & -3 \\ 2 & 0 & 3 \end{bmatrix}$$

i.e. find L and U (resp.) lower and upper triangle matrices such that $A = LU$.

b) Similarly, find the LUP decomposition of:

$$B = \begin{bmatrix} 1 & 2 & -2 \\ -2 & -4 & 6 \\ 0 & 4 & 2 \end{bmatrix}$$

i.e. find L and U (resp.) lower and upper triangle matrices, P permutation matrix (needed if the pivot is zero) such that $B = LUP$.

Ex 2.

a) Write a program that computes the LU decomposition of a matrix A.

b) Use the program to find numerically the LU decomposition of the 10×10 tridiagonal matrix:

$$A = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}.$$

Ex 3.

Find a symmetric definite matrix A that is *not* diagonal dominant.

Hint: a symmetric matrix $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ *is definite positive iff* $a > 0$ *and* $ad - b^2 > 0$.

Remark: we will see that a symmetric matrix A that is also strictly diagonal dominant is always definite positive (using Gershgorin theorem).

Ex 4.

Suppose that A is a symmetric definite positive matrix and M is a non-singular matrix of the same size. Show that MAM^T is also a symmetric definite positive matrix.

Ex 5.

Suppose $A = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{n1} & & & a_{nn} \end{bmatrix}$ symmetric definite positive matrix.

a) Consider M_1 the first row operation in the Gauss elimination:

$$M_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ -a_{21}/a_{11} & 1 & & \\ \vdots & & \ddots & \\ -a_{n1}/a_{11} & & & 1 \end{bmatrix}$$

Show that:

$$M_1 A M_1^T = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & \tilde{A} & \\ 0 & & & \end{bmatrix}$$

and that $\tilde{A}$ is a symmetric definite positive matrix (use **Ex. 4**).

b) By iteration, we construct a sequence of lower triangle matrices such that:

$$M_k \dots M_1 A M_1^T \dots M_k^T = D$$

with D diagonal matrix.

- Deduce that there exists L, lower triangle matrix, satisfying $A = LDL^T$.
- Show that there exists C, lower triangle matrix, such that: $A = CC^T$. The matrix C is called the Cholesky decomposition of A.

Extra Find numerically the Cholesky decomposition of A in **Ex. 2**.