

# MAT 423: Homework 5 (09/26)

**Ex 1.** Compute the norms  $\|\cdot\|_1$ ,  $\|\cdot\|_\infty$  and the spectral radius of the following matrices:

$$A = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

**Ex 2.**

Find two matrices  $A$  and  $B$  such that:

$$\|A\|_\infty < \|A\|_1 \quad \text{and} \quad \|B\|_1 < \|B\|_\infty.$$

**Ex 3.**

- Show that the application:  $\varphi(A) = |\det(A)|$  is not a norm.
- Prove that  $\varphi(A) = \max_{1 \leq i, j \leq n} |a_{ij}|$  is a norm but not a matrix norm.

**Ex 4.**

- Write a function that implement the Jacobi method:
  - input: a square matrix  $A$ , two vectors  $b$  and  $x^0$ ,  $M$  the number of iterations.
  - output: the final vector  $x^M$  and the sequence  $\text{error}(n) = \|Ax^n - b\|$  for  $0 \leq n \leq M$  (using the norm  $\|\cdot\|$  you prefer).
- Use the Jacobi method to solve the linear system:  $Ax = b$  with  $A$  the  $10 \times 10$  tridiagonal matrix:

$$A = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}$$

and  $b = (1, \dots, 1)^T$ .

- Estimate the convergence rate of the method, i.e.  $\text{error}(n) \approx C \cdot k^n$ .

*Extra*) Perform the same analysis with the Gauss-Seidel method.