MAT 423: Homework 10 (11/26)

Ex 1.

Consider the following minimization problem: minimize $J(x_1, x_2, x_3) = x_1 + 2x_2 + 2x_3$ over the domain Ω:

$$\begin{aligned} &\mathbf{x} \geq 0, \\ &x_1 + 2x_2 + 4x_3 \geq 1 \\ &4x_1 + 2x_2 + x_3 \leq 3 \end{aligned}$$

a) Introduce the slack variables and find a corner.
Deduce the optimal solution $\mathbf{x}_*$ using the simplex method.

b) Compute the dual problem.
Find all the corners and deduce the optimal solution $\mathbf{y}_*$

Ex 2.

Consider the minimization problem $J(\mathbf{x}) = \langle \mathbf{c}, \mathbf{x} \rangle$ with $\mathbf{c} = (1,1,1,3)^T$ with the constraint

$$A\mathbf{x} \geq \mathbf{b} \qquad \text{with} \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = (1,1,1,1)^T.$$

a) Show that $\bar{\mathbf{x}} = (1,1,1,0)^T$ and $\bar{\mathbf{y}} = (1,1,0,1)^T$ are feasible for (respectively) the primal and dual problem.

b) Show that $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal.
Hint: compute $\langle \mathbf{c}, \bar{\mathbf{x}} \rangle = \langle \bar{\mathbf{y}}, \mathbf{b} \rangle$.

Ex 3.

Suppose that all entries of A, $\mathbf{b}$, and $\mathbf{c}$ are positive. Show that both the primal and the dual are feasible.

Ex 4.

Suppose $\mathbf{x}_*$ is the unique optimal solution of a primal problem. Explain why if $\mathbf{c}$ is changed a little, then $\mathbf{x}_*$ still remains the optimal solution.