MAT 342: Homework 2 (09/07)

1 Section 1.2

Ex. 6)

$$
\begin{aligned}
&a)\,(0,1)\\
&b)\,\{\frac{3}{4}-\frac{5}{8}\alpha,\ -\frac{1}{4}-\frac{1}{8}\alpha,\ \alpha,\ 3\}\text{ with }\alpha\text{ real}\\
&c)\,\{0,\ \alpha,\ \beta\}\text{ with }\alpha,\beta\text{ real}\\
&d)\,\{-\frac{4}{3}\alpha,\ 0,\ \frac{1}{3}\alpha,\ \alpha\}\text{ with }\alpha\text{ real.}
\end{aligned}
$$

Ex. 9)

a) The linear system cannot be inconsistent since $(0, 0, 0)$ is always a solution (no matter what β is).

b) Using Gauss-Jordan elimination yields to:

$$
\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & \beta-2 & 0 \end{array}\right].
$$

Thus, the system has infinitely many solutions if $\beta - 2 = 0$, in other words $\beta = 2$.

Ex. 15) 3pts

We have to solve the linear system:

$$
\begin{aligned}
x_1 + 380 &= x_2 + 430\\
x_2 + 540 &= x_3 + 420\\
x_3 + 470 &= 420 + 400\\
420 + 450 &= x_1 + x_4.
\end{aligned}
$$

1pt

Thus,

$$
\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \\ 1 & 0 & 0 & 1 & 870 \end{array}\right]
$$

1pt

Solving this linear system gives:

$$x_1 = 280,\ x_2 = 230,\ x_3 = 350,\ x_4 = 590.$$

1pt

2 Section 1.3

Ex. 2)

$$a)\begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix} \; b)\,\text{Not possible,} \; c)\begin{bmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{bmatrix} \; d)\begin{bmatrix} 36 & 10 & 56 \\ 10 & 3 & 16 \end{bmatrix}.$$

Ex. 9) 3pts

a) $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$ 1pt

b) $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2 = A\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Thus, $\mathbf{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is a solution. The system does not have another solution since the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are linearly independent. 1pt

c) $\mathbf{c} = -\frac{5}{2}\mathbf{a}_1 - \frac{1}{4}\mathbf{a}_2$. 1pt

Remark. To obtain in a systematic way the coefficients in front of $\mathbf{a}_1$ and $\mathbf{a}_2$, one simply has to solve the linear system $A\mathbf{x} = \mathbf{c}$:

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -2 & -2 \end{array}\right].$$

Ex. 11)

There are at least 2 solutions:

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{y} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix},$$

since for instance $\mathbf{a}_1 + \mathbf{a}_2 = A\mathbf{x}$. We deduce that there are infinitely many solutions. More precisely, $\mathbf{x} + \alpha(\mathbf{y} - \mathbf{x})$ is a solution for any real number α:

$$\begin{aligned} A\big(\mathbf{x} + \alpha(\mathbf{y} - \mathbf{x})\big) &= A\mathbf{x} + \alpha(A\mathbf{y} - A\mathbf{x}) \\ &= \mathbf{b} + \alpha(\mathbf{b} - \mathbf{b}) = \mathbf{b}. \end{aligned}$$

Ex. 13) 4 pts

a) The set of solutions is given[1] by (replacing x_2, x_4, x_5 by resp. α, β, γ):

$$(-2 - 2\alpha - 3\beta - \gamma,\ \alpha,\ 5 - 2\beta - 4\gamma,\ \beta,\ \gamma)$$ 2pts

for any α, β, γ

b) Suppose $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$ is a solution, then:

$$\mathbf{b} = A\mathbf{x} \quad \Rightarrow \quad \mathbf{b} = \mathbf{a}_1\,x_1 + \mathbf{a}_2\,x_2 + \mathbf{a}_3\,x_3 + \mathbf{a}_4\,x_4 + \mathbf{a}_5\,x_5.$$

Since we only know $\mathbf{a}_1$ and $\mathbf{a}_3$, to find $\mathbf{b}$ we have to take a solution of the form: $\mathbf{x} = (x_1,\ 0\,x_3,\ 0,\ 0)$.

Thus, let's take $\alpha = \beta = \gamma = 0$, we have $\mathbf{x} = (-2,\ 0,\ 5,\ 0,\ 0)$. Therefore:

$$\mathbf{b} = \mathbf{a}_1 \cdot (-2) + \mathbf{a}_3 \cdot 5 = \begin{pmatrix} 8 \\ -7 \\ -1 \\ 7 \end{pmatrix}.$$ 2pts

[1] 5 unknowns and 2 equations, thus 3 degrees of freedom remaining