MAT 342: Homework 4 (09/20)

1 Section 2.3

Ex. 1)

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, $\det(A) = -7$, $\mathrm{adj}(A) = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/7 & 2/7 \\ 3/7 & -1/7 \end{bmatrix}$.

b) $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, $\det(A) = 10$, $\mathrm{adj}(A) = \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 4/10 & -1/10 \\ -2/10 & 3/10 \end{bmatrix}$.

c) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{bmatrix}$, $\det(A) = 3$, $\mathrm{adj}(A) = \begin{bmatrix} -3 & 5 & 2 \\ 0 & 1 & 1 \\ 6 & -8 & -5 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -1 & 5/3 & 2/3 \\ 0 & 1/3 & 1/3 \\ 2 & -8/3 & -5/3 \end{bmatrix}$.

d) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $\det(A) = 1$, $\mathrm{adj}(A) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = A^{-1}$.

Ex. 6)

Since $A \cdot \mathrm{adj}(A) = \det(A) \cdot I$, then when A is singular (i.e. $\det(A) = 0$), $A \cdot \mathrm{adj}(A)$ is the zero matrix.

2 Section 3.1

Ex. 13)

We show that the space $\mathbb{R}$ with the rule $\otimes$ does not satisfy the axiom $A4$ (i.e. there is no element "$-x$"). Indeed take $x = 1$ in $\mathbb{R}$, for any element y in $\mathbb{R}$, we have:

$$1 \otimes y = \max(1, y) \geq 1 \neq 0.$$

Thus, one cannot have $1 \otimes y = 0$. Therefore, $\mathbb{R}$ with the "addition" $\otimes$ is not a subspace.

3 Section 3.2

Ex. 2)

a) **No**. The zero vector is not in the subset.

b) **Yes**. It is actually a line generated by: $\text{Span}\big((1,1,1)\big)$.

c) **Yes**. It is a plan orthogonal to the vector $(1,1,-1)$.

c) **No**. Take $\mathbf{u} = (1,0,1)$ and $\mathbf{v} = (0,1,1)$ both in the set. The sum $\mathbf{u}+\mathbf{v} = (1,1,2)$ does not belong to the subset anymore.

Ex. 4) 4pts

Solving the problem $A\mathbf{x} = \mathbf{0}$ leads to:

a) $N(A) = \left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}$. $\boxed{1\text{ pt}}$

b) $N(A) = \left\{\begin{pmatrix} -2\alpha+3\beta \\ \alpha \\ \beta \\ 0 \end{pmatrix}, \quad \text{with } \alpha, \beta \text{ scalars}\right\}$. $\boxed{1\text{ pt}}$

c) $N(A) = \left\{\begin{pmatrix} \alpha \\ \alpha \\ \alpha \end{pmatrix}, \quad \text{with } \alpha \text{ scalar}\right\}$. $\boxed{1\text{ pt}}$

d) $N(A) = \left\{\begin{pmatrix} -5\alpha-\beta \\ \beta \\ -3\alpha \\ \alpha \end{pmatrix}, \quad \text{with } \alpha, \beta \text{ scalars}\right\}$. $\boxed{1\text{ pt}}$

Ex. 10) 3pts

a) S_1 subspace: $\boxed{1\text{ pt}}$

- Let $B \in S_1$ and α scalar: $\alpha BA = \alpha(BA) = 0$.
  Thus, $\alpha B \in S_1$.
- Let $B_1, B_2 \in S_1$: $(B_1+B_2)A = B_1A + B_2A = 0$.
  Thus, $B_1 + B_2 \in S_1$.

Therefore, S_1 subspace.

b) S_2 is not a subspace. Take $B \in S_2$ and $\alpha = 0$, then: $A(0\cdot B) = (0\cdot B)A = 0$. Thus, $0 \cdot B$ not in S_2. $\boxed{1\text{ pt}}$

c) S_3 subspace: $\boxed{1\text{ pt}}$

- Let $B \in S_3$ and α scalar: $A(\alpha B) + \alpha B = \alpha(AB + B) = 0$.
  Thus, $\alpha B \in S_3$.
- Let $B_1, B_2 \in S_3$: $A(B_1 + B_2) + (B_1 + B_2) = AB_1 + B_1 \, + \, AB_2 + B_2 = 0$.
  Thus, $B_1 + B_2 \in S_3$.

Therefore, S_3 subspace.

Ex. 13) 3pts

a) We try to solve $\mathbf{x} = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$ which leads to:

$$\left[\begin{array}{rr|r} -1 & 3 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 6 \end{array}\right]$$

$\boxed{\text{1 pt}}$

This problem does not have a solution, thus $\mathbf{x}$ not in $\mathrm{Span}(\mathbf{x}_1, \mathbf{x}_2)$.

$\boxed{\text{.5 pt}}$

b) Similarly, solving $\mathbf{y} = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$ leads to:

$\boxed{\text{.5 pt}}$

$$\left[\begin{array}{rr|r} -1 & 3 & -9 \\ 2 & 4 & -2 \\ 3 & 2 & 5 \end{array}\right]$$

We find out that $c_1 = 3$ and $c_2 = -2$. Thus, $\mathbf{y} \in \mathrm{Span}(\mathbf{x}_1, \mathbf{x}_2)$.

$\boxed{\text{1 pt}}$