MAT 342: Homework 7 (10/18)

1 Section 5.2

Ex. 2)

a) We need to find vectors $\mathbf{u} = (x_1, x_2, x_3)$ such that $\langle \mathbf{u}, \mathbf{x} \rangle = 0$ with $\mathbf{x} = (1, -1, 1)$, in other words:

$$x_1 - x_2 + x_3 = 0.$$

We have for instance: $\mathbf{u}_1 = (1, 1, 0)$ and $\mathbf{u}_2 = (0, 1, 1)$. The two vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ are linearly indepedent. Moreover, $S^\perp$ is a subspace of $\mathbb{R}^3$ of dimension 2:

$$\dim(S) + \dim(S^\perp) = \dim(\mathbb{R}^3) \quad \Rightarrow \dim(S^\perp) = 2.$$

Thus, $\mathbf{u}_1$ and $\mathbf{u}_2$ form a basis of $S^\perp$.

b) S is a line and $S^\perp$ is a plane.

Ex. 6)

Suppose $\mathbf{u} = (3, 1, 2)$ is a row vector of A, i.e. a column vector of A^T. Thus, $\mathbf{u} \in \mathrm{Im}(A^T)$. We deduce that $\mathbf{u} \perp N(A)$, in other words for any $\mathbf{v} \in N(A)$, we must have: $\langle \mathbf{u}, \mathbf{v} \rangle = 0$. Now, if $\mathbf{v} = (2, 1, 1) \in N(A)$, then:

$$\langle \mathbf{u}, \mathbf{v} \rangle = 6 + 1 + 2 = 9 \neq 0.$$

Contradiction.

2 Section 5.3

Ex. 3) 2pts

a) $A^T A\mathbf{x} = A^T\mathbf{b} \;\Rightarrow \begin{bmatrix} 6 & 12 \\ 12 & 24 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \end{pmatrix}.$

The solutions are given by:

$$c_1 = 1 + 2\alpha, \qquad c_2 = -\alpha, \qquad \boxed{\text{1 pt}}$$

with α scalars.

b) $A^T A\mathbf{x} = A^T\mathbf{b} \;\Rightarrow\; \begin{bmatrix} 3 & 0 & 6 \\ 0 & 14 & 14 \\ 6 & 14 & 26 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ 24 \end{pmatrix}.$

The solutions are given by:

$$c_1 = 2 - 2\alpha, \quad c_2 = 1 - \alpha, \quad c_3 = \alpha,$$ 1 pt

with α scalars.

Ex. 4) 2pts

a) $\mathbf{p} = A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Moreover, $A^T(\mathbf{b} - \mathbf{p}) = 0$, thus $\mathbf{b} - \mathbf{p}$ is orthogonal to the column vectors of A. 1 pt

b) $\mathbf{p} = A\mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$. Moreover, $A^T(\mathbf{b} - \mathbf{p}) = 0$, thus $\mathbf{b} - \mathbf{p}$ is orthogonal to the column vectors of A. 1 pt

Ex. 5) 3pts

a) Linear fitting: $\mathbf{y} = 1.8 + 2.9\mathbf{x}$ 2 pt

b) See figure 1. 1+1 pt

Ex. 6) 3pts

A quadratic fitting: $\mathbf{y} = c_1 + c_2\mathbf{x} + c_3\mathbf{x}^2$ leads to the linear system:

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 9 \end{pmatrix}$$

The least squares solution is:

$$c_1 = .55, \quad c_2 = 1.65, \quad c_3 = 1.25.$$

Thus, $\mathbf{y} = .55 + 1.65\mathbf{x} + 1.25\mathbf{x}^2$.

2 pt

3 Section 5.4

Ex. 16)

$$\|\mathbf{x} - \mathbf{y}\|_1 = 5, \qquad \|\mathbf{x} - \mathbf{y}\|_2 = 3, \qquad \|\mathbf{x} - \mathbf{y}\|_\infty = 2.$$

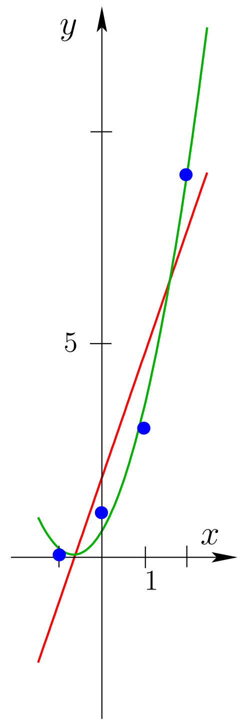

Figure 1: The data set $(\mathbf{x}, \mathbf{y})$ (blue) with the linear fitting (red) and the quadratic fitting (green).